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Total Number of Pages: 03

MCA
MCC305

3rd Semester Regular/ Back Examination – 2015-16

PROBABILITY AND STATISTICS

BRANCH(S): MCA

Time: 3 Hours

Max Marks: 70

Q.Code:T637

Answer Question No.1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

Use of statistical tables is allowed

- Q1 Answer the following questions: (2 x 10)
- State Bayes' theorem of conditional probability.
 - Find the probability distribution of the random variable X representing the outcome when a single die is rolled once.
 - What is the relationship between density function and distribution function of a random variable?
 - A coin is biased so that head is three times as likely to occur as tail. Find the expected value of the tails when the coin is tossed twice.
 - In a certain factory the probability of an accident on any given day is 0.005. What is the probability that in a given period of 200 days, there shall be an accident on a day.
 - What is Weibull distribution? In which situation Weibull distribution is more appropriate to use?
 - Explain the principle of maximum likelihood estimation.
 - What is prediction interval? What is the use of prediction interval?
 - Explain different errors in testing of hypotheses.
 - Explain simple linear regression model.
- Q2 (a) The probability that an automobile being filled with gasoline will also need an oil change is 0.25; the probability that it also need a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14. Find (5)
- If the oil had to be changed, what is the probability that a new oil filter is needed?
 - If the new filter is needed, what is the probability that the oil has to be changed?
- (b) In a certain prison it is known that $\frac{2}{3}$ of the inmates are under 25 years of age. It is also known that $\frac{3}{5}$ of the inmates are male and that $\frac{5}{8}$ of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is a female with at least 25 years of age. (5)
- Q3 (a) The proportion of the budgets for a certain type of industry that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of the proportion is given by (5)

$$f(y) = \begin{cases} 5(1-y)^4 & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Verify that the above is a valid density
- (ii) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution control?

- (b) The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. The joint density function of these variables is (5)

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Determine if X and Y are independent
- (ii) Find $P(1/4 < X < 1/2 \mid Y = 3/4)$

Q4 (5)

- a) On a laboratory assignment, if the equipment is working, the density function of the outcome X is

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and variance of X

- b) State Chebyshev theorem. Show that the Chebyshev theorem holds for $k = 2$ and $k = 3$ for the random variable X whose density function is given by (5)

$$f(x) = \begin{cases} \frac{1}{5} & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Q5 a) The company generally purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units. (5)

- (i) What is the probability of rejecting a lot that is 1% defective?
- (ii) What is the probability of accepting a lot that is 5% defective?

- b) A company pays its employees an average wage of Rs15.90 an hour with a standard deviation of Rs1.50. If the wages are approximately normally distributed, find (5)

- (i) What percentage of workers receives wages between Rs13.75 and Rs16.22 an hour?
- (ii) The highest 5% of the employee hourly wages is greater than what amount?

Q6 a) The study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds. (5)

- (i) What is the probability that the response time exceeds 5 seconds?
- (ii) What is the probability that the response time lies between 5 seconds to 10 seconds?

- b) Find the probability that a random sample of 25 observations from a normal population with variance $\delta^2 = 6$ will have a variance s^2 (5)

- (i) greater than 9.1
- (ii) between 3.462 and 10.745

Q7 a) A machine is producing metal pieces which are cylindrical in shape. A sample of pieces is taken and the diameters are 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01 and 1.03 centimeters. Find a 99% confidence interval for the mean diameter of pieces from the machine, assuming an appropriate normal distribution. (5)

b) Consider a random sample $x_1, x_2, x_3, \dots, x_n$ from a normal distribution $N(\mu, \delta)$. Find the maximum likelihood estimators for μ and δ . (5)

Q8 (a) Test the hypotheses that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal (5)

(b) The grades of a class of 9 students in midterm examination (x) and final examination (y) are as follows (5)

x:	77	50	71	72	81	94	96	99	67
y:	82	66	78	34	47	85	99	99	68

Estimate the regression lines.